Probability

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What is Probability?

- **Probability** is an interesting branch of mathematics that is widely used in genetics, insurance, finance, medicine, sociological surveys, marketing and science.
- We use probability to measure the **chance** or **likelihood** of an event or events occurring in the future.

What's an Event?

- An **event** is something that may or may not occur at some time or during some period in the future.
- We talk about events in terms of chances
  - "I will **probably** play tennis this summer"
  - "It isn't **likely** that Britney Spears will win another Grammy"
  - "My **chances** of acing this probability test is excellent!"

How do we describe future trends?

- What words can we use to describe an expected occurrence?
  - Certain
  - Probable
  - Fifty-fifty
  - Improbable
  - Impossible
- These tell us approximately how the event is expected to occur; but they're vague
- Probability allows us to better quantify the chance of an event occurring.

Probability Scale: Impossible!

- What are some impossible events?
- What percent of the time can they happen?
- What number represents the probability of something that can NEVER HAPPEN? ___

0

Probability Scale: For Sure!

- What are some events that are certain to occur?
- Will they happen? What percent of the time? ____
- If an event is **certain** to occur, its probability is ____?

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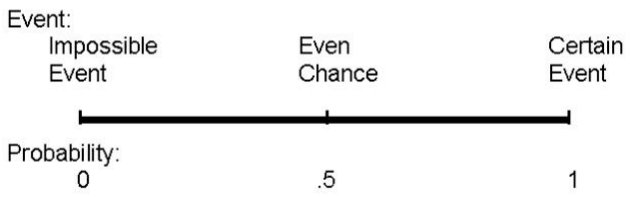
The Probability Scale



The Probability Scale

- What happens between 0 and 1 in the scale?
- What are 3 examples of something having a probability of .5?
- What's an example of a probability between 0 and 1 that is NOT .5?
  - Ex. The weatherman says tomorrow there is a 20% chance of rain. What is the probability for it not to rain tomorrow? _____

Probability Experiment

- A **probability experiment** involves performing a number of **trials** to enable us to measure the chance of an event occurring in the future.
- A **trial** is a process by which an **outcome** is noted.
  - Ex. Rolling a die, picking a card, spinning a spinner

What Could Happen?

- The **sample space** of an experiment is the set of all possible outcomes of any trial of the experiment to be conducted.
- Ex: if a coin is tossed, then the two possible outcomes are _____ and _____.
- The set of all possible outcomes is therefore {*H*, *T*}. This is called the sample space of the experiment and is denoted by *S*.
  S=_____

Rolling a number cube

- Our **experiment** is to roll a cube 60 times & record the outcome
- The desired **event** is to roll a six
- We call each individual roll a **trial**
- How many trials will we do? ____
- What are the **outcomes** of a roll? ___

Let's Do It!

- How many times did we successfully roll 6?__
- How many times did we roll total? ____
- Formula for Probability:

$$P(\text{event}) = \frac{\#\,successes}{\#\,total}$$

- So in this experiment our probability is: _____

Experimental Probability

- If I do this same experiment with each of my classes, will they have the same probability?
- A measure of probability based off real-life events is termed **experimental probability**.
- What other kind of probability exists?

Theoretical Probability

- **Theoretical probability** is what <u>should</u> happen, what we can predict what will happen in real-life
  - What is the theoretical probability for flipping a coin and landing on heads? ___
  - What about rolling a number cube? What is the theoretical probability for rolling an even number?

Let's go back to our roll!

In our experiment, we found experimental probability to be _____.

What would the theoretical probability be to roll a 6: _____.

Are they different or the same and why?

_____

_____

Lots and Lots of rolls!

- Now we will roll 600 times. Will our probability change? Why? _____

_____

- Our theoretical probability for 60 rolls: ____
- Our experimental probability for 60 rolls: ___
- Our probability for 600 rolls: _____
- What do we notice?_____

Law of Large Numbers

- Law of Large Numbers says that the more times you do something, the closer you will get to what is supposed to happen
- The more simulations we conduct the closer our experimental probability will get to the theoretical probability
- In statistics, it means that the larger sample size you use the closer your sample will represent the entire population.

Gambler's Fallacy

- If you roll a 6 first, does that mean you will have less probability to roll a 6 the next time you roll? _____
- Each roll is independent of one another, so every time any person in the world rolls a number cube they have equal probability to roll a six.
- So, an adult who pulls a slot machine, what's going on there?

Gambler's Fallacy Cont.

- A pull of the slot machine's handle is completely independent of previous pulls.
- The slot machine has no memory of what has come before.
- You might play a slot machine for 2 weeks without hitting the big jackpot, and someone else can walk in and hit it in the first 5 minutes of play.
- Since each event is independent, probability is equivalent each day an adult would play.

All about cards

- A pack of 52 playing cards consists of four suits:
  - clubs, spades, diamonds and hearts
- Each suit has 13 cards which are:
  - Number cards: 2, 3, 4, 5, 6, 7, 8, 9, 10
  - Face cards: jack, queen, king, and ace.
- Clubs and spades are black
- Diamonds and hearts are red
  - 26 red cards and 26 black cards

Now let's play:

- Find the probability of drawing from a well-shuffled pack of cards:
  a. a black card
  b. the queen of diamonds
  c. a king
- Answers :
  a. P(a black card)= _____
  b. P(the queen of diamonds)= _____
  c. P(a king)= _____

Let's do a useful one:

- On the hit television show "Endurance", (a teen version of "Survivor"), teams who are in danger of leaving the island play one final game to determine their fate:

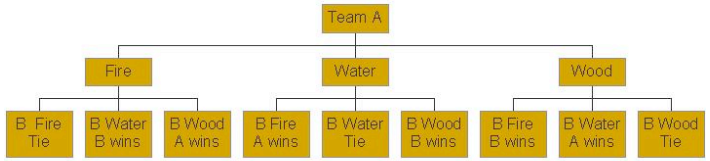
Rock, Paper, Scissors

Probability of RPS

- Of course, on "Endurance" they play

  Fire, Water, Wood
- If you and your partner are playing, which element would you choose?
- Is there a strategy?
- Why would a strategy exist in a game?

Finding Probability of RPS



Probability of RPS

- For any given choice of water, wood, or fire, does team A have an advantage over team B?
- What if team A would automatically win anytime a tie occurred. Does this change team A's probability to win?
- Would this be a fair game?

"I Have, Who Has" Cards

I have: Let's Begin! Who has: The term we used to describe the likeliness or chance of an event in the future?	I have: Probability Who has: The probability to roll a 4 on a number cube?
I have: $\frac{1}{6}$ Who has: Something that may or may not occur at some time in the future?	I have: An event Who has: The probability something will certainly occur?

I have: 1 Who has: The set of all possible outcomes of any trial of an experiment?	I have: Sample space Who has: The probability in decimal form for a quarter to flip heads?
I have: .5 Who has: The process by which an outcome is noted?	I have: A trial Who has: The number of outcomes there are on a standard number cube?

I have: 6 Who has: The name for all the possible things that can occur in an experiment?	I have: Outcomes Who has: The formula for standard probability?
I have: $P(\text{event}) = \frac{\#\text{ of successes}}{\text{total}\#}$ Who has: The word describes the event where a cow jumps over the moon?	I have: Impossible Who has: The probability of an impossible event?

I have: 0 Who has: The probability of drawing a red four from a deck of cards?	I have: $\frac{2}{52} or \frac{1}{26}$ Who has: The type of probability is based off what should happen?
I have: Theoretical Probability Who has: The probability of drawing a black face card?	I have: $\frac{8}{52} or \frac{2}{13}$ Who has: The type of probability which is based off real–life events?

I have: Experimental Probability Who has: The probability of winning a standard game of Rock, Paper, Scissors?	I have: $\frac{3}{9} or \frac{1}{3}$ Who has: The rule that says the more times you do something the closer you will get to what's supposed to happen?
I have: The Law of Large Numbers Who has: The number of outcomes that exists if a person rolls a number cube and flips a coin?	I have: 8 Who has: The probability in fraction form for rolling an even number on a die?

I have: $\frac{1}{2}$ Who has: Idea that life will "even out" without regard to individual probability per event?	I have: Gambler's Fallacy Who has: An interesting way to play Rock, Paper, Scissors?
I have: Fire, Water, Wood Who has: Probability of drawing the Queen of Spades?	I have: $\frac{1}{52}$ Who has: The probability of it **not** raining if the news says the chance of rain is 75%?

I have: .25 Who has: Percent of chance for rolling a number greater than 4 on a 10–sided number cube from (0–9)?	I have: 50% And that's a wrap! Hooray!!